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Algebra Tiles Effect on Mathematical Achievement of Students with Learning Disabilities

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Abstract

Decreasing the mathematical learning gap between students with disabilities and their neuro-typical peers has been a priority for educators and researchers for years. Teachers require an easily implemented intervention to improve students' comprehension of algebraic concepts. Algebra tiles are an intervention that has been frequently used to improve students' understanding of abstract math concepts. This study used a quasi-experimental design with a pre- and post-test to examine the impact of algebra tiles on students' understanding of distributive property and evaluating expressions. The treatment group received direct instruction and algebra tiles for a five-week period. Independent and paired samples t-tests were conducted to determine if there were statistically significant differences between the means of both groups on the post-test. The results of this study suggest that there were no significant differences in the final achievement of both groups. However, both group's scores improved from pre- to post-test with statistically significant scores. Therefore, although using algebra tiles increased students' scores they were not statistically more beneficial than direct instruction. Future studies should continue looking for ways that will allow student with disabilities access to the general education curriculum.

Keywords: learning disabilities, mathematics, algebra tiles

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Literature Review

Schools and jobs have increasingly put more emphasis on the need for their prospects to be skilled in algebraic concepts. Persons who desire to operate with success in the world must achieve understanding of certain mathematical concepts such as algebra (Cass, Cates, Smith, & Jackson, 2003; Fogen, 2008). However, studies have shown that many students are unable to access the algebra curriculum and thus are left behind and get stuck in a revolving door of lower level classes (Maccini & Hughes, 2000; White, Porter, Gamoran, & Smithson, 1997). Students with disabilities are at a higher risk for struggling with algebraic concepts and experience problems with the processes involved (Maccini & Hughes, 2000). For this reason, it is crucial for these students to be taught in ways that play to their strengths and capabilities. Strategies and best practices when teaching students with disabilities should be individualized and research based (Van de Walle, Karl, & Bay-Williams, 2016).

According to Individuals with Disabilities Education Act (IDEA; 2004) an Individualized Education Program (IEP) is a written statement developed for each child with a disability. It should include but is not limited to the child's present levels, annual goals, and services offered. Students who qualify for an IEP are at a greater risk for not being successful in an algebra class compared to their general education peers (Foegen, 2008; Maccini & Ruhl, 2000). This is due in part to the fact that many students with disabilities have complications applying problem solving strategies (Maccini & Hughes, 2000; Steele & Steele, 2003). It has been suggested that 5% to 8% of school-aged children have some type of mathematical learning disability (Satsangi, Bouck, Taber-Doughty, Boefferding, & Roberts, 2016).

A learning disability (LD) is recognized as a processing disorder that hinders the student's capability to achieve academic standards that is not directly related to other disabilities (Steele & Steele, 2003). Children with LD often score below or far below proficiency in mathematics testing, especially in algebra (Foegen, 2008). Many of the characteristics related to learning disabilities directly correlate with a student's ability to understand and solve algebra problems (Steele & Steele, 2003). According to Strickland and Maccini (2012) 91% of 8th graders and 94% of 12 graders with LD scored below proficiency on standardized math tests. This is very concerning as the performance gap between typical students and students with LD continues to grow. As this gap widens it will limit access to higher-level education and skilled jobs for students who suffer from these disabilities.

The ultimate goal of secondary education is to provide students with the tools they will need to be successful in the real world. When students prepare for life after high school, they must meet ever increasing requirements in school mathematics as schools strive to produce college and career ready pupils (Strickland & Maccini, 2012). Students are often required to pass an algebra class in order to receive a high school diploma, this task can be very challenging for students with disabilities (Gagnon & Maccini, 2001). Mathematic accomplishments have been strongly linked to a person's access to higher education and higher income (Foegen, 2008). It is pertinent that schools find different ways to bridge the attainment divides so that students with disabilities are capable of obtaining the same quality of life as their neuro-typical peers.

Unfortunately, students with disabilities continue to be passed up as the achievement disparity between them and their typical functioning peers has seen little improvement (Maccini et al., 2008). This may be due to the notion that students with LD typically have poor conceptual understanding of many aspects of mathematics (Geary, 2004). This population of students can have "visual-, auditory-, and motor-processing problems, memory deficits, language disabilities, weak abstract-reasoning skills, and social and behavioral concerns" (Steele & Steele 2003). They typically struggle with their working and long-term memory that is central in successful completion of an algebra course (Statsangi et al., 2016). Students with LD are at greater risk to commit more errors and are prone to use more cognitively immature strategies when solving problems (Geary, 2004). The combination of poor strategies, difficulties with working memory and basic mathematical operations leads to great struggle with the algebra curriculum for students with LD (Maccini & Hughes, 2000). It is essential that instructional strategies be established that will expedite acquirement, continuation and generalization of math competence for students with LD (Cass et al., 2003). One strategy that has been described as an effective approach to improve student performance is the use of manipulatives (Carbonneau, Marley, Selig, 2013). Manipulatives are a tool that can be used to help students acquire understanding of mathematical concepts by motivating and stimulating a student's senses.

Manipulatives and Mathematics

Manipulatives are concrete objects that students can physically arrange or operate in some way to represent a variety of mathematical relationships (Cass et al., 2003). They can be specifically designed for mathematical purposes such as geoboards, fraction and algebra tiles, or not designed for mathematics such as buttons, straws or toothpicks. The choice of manipulative is not what has an impact on performance, rather the impact comes from instruction being differentiated and students being allowed to create their own relationships with the algebra content (Ojose, 2008).

The National Council of Teachers of Mathematics (NCTM, 2000) suggest that to help students meet mathematical standards, educators must strategically integrate the use of concrete manipulates into everyday classroom instruction. The NCTM (2000) also calls for the use of hands-on exploration of the mathematical standards in order to benefit student understanding and problems solving. Through the use of these types of hands-on learning and the experiences students create, they are able to make abstract ideas concrete (Ojose, 2008). It is important to use a variety of modalities, such as manipulatives, to differentiate instruction to meet the needs of all students. With the use of concrete hands-on activities students are not only able to comprehend abstract mathematical concepts, but are also able to create positive mathematical experiences (Furner, Yahya, & Duffy, 2005).

Manipulatives can aid in student retention of topics as well as student buy-in to the curriculum (Allsop, 1999). The use of these concrete items not only can help to motivate students by making math fun, but they can help stimulate students to think mathematically (Herbert, 1985). When students are having fun and engaged it allows teachers to give them the ability to be creative and be active during their learning. Teachers no longer need to rely on worksheets for student assessment and practice but rather can incorporate fun and engaging learning into their repertoire with the use of manipulatives (Furner et al., 2005). By using the manipulatives during assessment teachers can gauge if students can apply the knowledge to real world situations. Students are then able to use different resources to solve problems and enjoy their math time (Herbert 1985; Kamina & Iyer, 2009).

The idea of manipulatives is rooted in the constructivist theory of Jean Piaget (Ojose, 2008). The use of concrete manipulatives can promote content mastery by aiding the growth of abstract reasoning (Carbonneau et al., 2013; Piaget, 1962). When students are allowed to use manipulatives they are putting meaning to something that they may have only seen as abstract. Their senses are stimulated, as they are able to learn kinesthetically and make tactile connections (McClung, 1998). Piaget's theory speaks to the idea that children should learn through their senses; and, that it is necessary for them to experience and manipulate the ideas that they are learning. Increased student engagement in the learning process can lead to higher achievement (Ross & Willson, 2012). It is no longer considered best practice to teach children using the same continued drill and practice model, especially with higher-level mathematics (Heddens 1986). Students cannot learn about mathematics by listening to a teacher lecture on it; that is, students benefit from actively participating in their learning. By engaging students in their learning through the use of manipulatives, educators can get students to become active players rather than passive bystanders. This active participation through the use of manipulatives allows students to learn by discovery rather than teacher direction (Carbonneau et al., 2013).

Furthermore, manipulatives provide students with the opportunity to apply their own meaning and experiences to the math they are learning. In addition, it may help to build the foundation needed for students to access more abstract teachings (Ojose, 2008). Educators have found that even the use of simple manipulatives have increased the learning of students in beginning algebra (Allsopp 1999). This is especially true for students with disabilities who have been shown to struggle with algebra concepts (Macinni & Ruhl, 2000). With the Common Core State Standards and the increasing need to understand abstract mathematics, teachers must move away from the status quo and begin implementing different styles of teaching to represent all students. Teachers need to move towards instruction that enables students with disabilities to break down the walls of difficult curriculum so that they may attain their goals.

Math manipulatives and students with LD. Students with LD exhibit serious issues when it comes to comprehending the different topics involved in the curriculum (Maccini & Hughes, 2000). To help alleviate some of these issues for students with LD, the use of concrete materials can enable students to remember the procedural steps and problem solve at a greater rate than without them (Witzel, 2005). Many studies have looked at the use of two specific instructional strategies that employ manipulatives and have been shown to increase the algebra comprehension of students with LD (Maccini et al., 2008; Maccini & Ruhl, 2000; Witzel, 2005). These strategies are Concrete-Representational-Abstract (CRA) and Concrete-Semi Concrete-Abstract (CSA), both are based on the same theories and follow the same procedures. The graduated instructional sequence includes a three-stage progression in which instruction progresses through concrete, semi-concrete and abstract stages (Macinni et al., 2008). Students first solve problems using hands-on learning with manipulatives, then they move to the semi-concrete stage in which they use visual representations to help with mathematical problems and lastly students enter the abstract phase in which they use numbers and symbols.

The majority of the literature written about manipulatives and students with LD involves the use of these strategies. The study of Maccini and Ruhl (2000) showed that through the use of algebra tiles paired with the CSA model students improved their ability to subtract integers. According to Maccini and colleagues (2008) the use of CSA can also help the success of students with special needs to better understand more abstract ideas of math. This is because students move through the different stages of learning allowing them to generalize their knowledge to the abstract ideas (Allsopp, 1999). This study will only be looking at the concrete stage as students will be able to use manipulatives throughout the entire study with no additional stages of instruction. The literature suggests that the use of CSA or CRA coupled with algebra tiles can increase the success rate of students with LD and the algebra curriculum (Maccini & Ruhl, 2000; Witzel, 2005).

Algebra tiles and algebra learning. Algebra tiles are double-sided rectangular figures that can be used to represent constants and variables. They are two different colors with one side/color representing a negative monomial and the other representing a positive monomial. Algebra tiles can be used throughout algebra instruction and are most commonly used to solve integer problems but can also be used with more complex ideas such as equations.

Algebra tiles have been shown to help students when it comes to algebraic concepts such as integer operations and evaluating equations and expressions (Chappell & Strutchens, 2001). It is critical for teachers to ensure they are guiding their students to make connections between the manipulative and abstract idea it is representing (Chappell & Strutchens, 2001). By using the algebra tiles students can label their own variable to each tile and use them to represent the conceptual ideas of expressions and integers. As students begin to make connections between the concrete and abstract using algebra tiles, they are able to go greater in depth on familiar topics, which can in turn lead to greater understanding (Chappell & Strutchens, 2001; Ojose, 2008).

In a study by Maccini and Ruhl (2000) students with LD increased their accuracy of subtraction of integers by using algebra tiles. Maccini and Hughes (2000) also found an increase in student knowledge of algebra concepts by allowing the use of algebra tiles during instruction. Using the CRA strategy on a group of students with LD Strickland and Maccini (2012) were able to demonstrate student learning and improvement when dealing with multiple linear expressions within area problems. In a meta-analysis by Carbonneau, Marley, and Selig (2013) only two out of the fifty-five studies that involved mathematics and manipulatives looked at students with disabilities. There is insufficient research and literature accessible on algebra instruction for students with disabilities. It is necessary to find instructional strategies to best promote student learning and success in mathematics for this population of students. This study looks to fill that gap of missing research to insure that best practices for students with disabilities is used.

Method

Purpose

The purpose of this research was to examine different ways educators can better instruct students with disabilities in algebra. Though there were many studies with mathematical instruction and the use of manipulatives at the primary levels with neuro-typical students, (Dawson, 1955; Fujimura, 2001; Peterson, Mercer, & O'Shea, 1988; Suh & Moyer, 2007) there were few studies with manipulatives and students with disabilities at the secondary level (Cass et. al, 2003, Chappell & Strutchens, 2001). This study investigated if the use of manipulatives (i.e., algebra tiles) could help students with disabilities in a secondary setting improve comprehension of algebra concepts.

Research Question

Does the use of algebra tiles increase the comprehension of algebra expression in secondary students with disabilities?

Hypothesis

Based on the research (Goins, 2001; Maccini & Ruhl 2000), the hypothesis of this study was that algebra tiles would increase student comprehension of algebraic expressions. According to Chappell and Strutchens (2001) when students use algebra tiles they can become more involved in their learning and put more meaning to the abstract teachings. This allows for greater understanding and the ability to make these abstract ideas concrete. In studies done by Macinni and Ruhl (2000) and Maccini and Hughes (2000) findings show that there can be an increase in students' abilities to solve algebraic problems through the use of manipulatives.

Research Design

This study implemented a quasi- experimental quantitative design. The two groups took a pre- and post-test to obtain student present level data on the topic. In between the pre- and post-test the treatment group received an intervention using algebra tiles. They used them during instruction as well as practice, while the control group did not have access to them. The control group continued with drill and practice of abstract ideas through the use of their text and worksheets.

Independent variable. The independent variable in this study were algebra tiles, they are two different colored rectangles with one side/color representing a negative monomial and the other representing a positive monomial. The tiles are used to represent a variety of variables and numbers (Chappell & Strutchens, 2001). Algebra tiles can be used throughout the different stages of algebra instruction and are often used when teaching area models as well as integer operations.

Dependent variable. Students' post-test scores on an algebra test that contained content that relates to algebraic expressions. There were seventeen problems on the post-test that ranged from order of operations to simplifying and evaluating variable expressions. The post-test scores were compared against the pre-test data and whether or not students' scores improved (Ross & Willson 2012; Witzel, 2005).

Setting & Participants

This study took place in a public high school on the central coast of California. Students were a part of the mild/moderate special education program and were in what is called Special Day Classes (SDC) for their core curriculum classes. Students were on track to receive a certificate of completion, not a diploma upon their graduation. The two classes were labeled

International Math 1 (M). These classes used modified curriculum that disqualified students from receiving algebra credit. Students received their primary math instruction in these classes, and all students fell below average or far below average on mathematics standardized testing. There was a total of 36 participants in this study all ranging in grades from freshman to juniors and their mathematical abilities ranged from two to eight grade levels below their current grade level. The participants were spread across two different classrooms with two different teachers, both teachers taught from the same curriculum and collaborated on instruction. This was a convenience sample as the treatment group consisted of the researcher's assigned classroom and the control group was students within the same Mild to Moderate program as the treatment group. Within that sample participants were purposefully selected as they all have learning disabilities.

Treatment group. The treatment class was a class of 19 students with nine females and ten males. A majority of the students speak Spanish at home and 95% of the students were classified as English Language Learners (ELL). Furthermore, 95 % of the students received free and reduced lunch. All of the students were a part of the mild to moderate SDC program and qualified for an IEP by LD.

Control group. The control group was a class of 17 students with eight females and nine males. Of the students, 85% of the students were ELL with Spanish being the language spoken at home and 85% of students received free and reduced lunch. Two students were a part of the Resource Program with added difficulties in mathematics and one student was a part of the Life Skills program with a splinter skill in math (i.e., capability of doing math specific tasks that do not generalize to other subjects). All of the students had an IEP and qualified for special education services under the categories of Specific Learning Disability or Autism.

Measures

Students took a pre-test and a post-test that were identical. This was the basis of the study and their scores were calculated from these tests. The test questions were scored either correct or incorrect with each question being worth one point. The test came directly from the student textbook (*Pacemaker: Algebra 1,* 2001*)* and had 17 questions. The study took five weeks from pre-test to post-test as students were assessed on their knowledge of algebraic expressions. The pre- and post-test evaluated student knowledge on their overall ability to solve problems involving algebraic expressions (see Appendix A and B). Individual test problems included combining like terms, distributive property, and evaluating for a variable.

Validity. The pre and post-test was used from the book titled *Pacemaker: Algebra 1* (2001). The test was used from the student's chapter of study during the experiment and was used to measure a student's ability to understand algebraic expressions.

Reliability. The researcher scored both tests for all students. The questions were worth one point and scored as either correct for one point or incorrect for zero. The same pre- and post-test was given to all of the participants. *Pacemaker: Algebra 1* (2001) teacher's edition provided the answers and thus it was clear if the answer was accurate or not. Test scores could increase due to familiarity to the same test, in order to prevent this question order was changed.

Intervention

Students were introduced to algebra tiles, a type of manipulative that aided in their acquisition of algebraic skills. Van de Walle and colleagues (2016) talk about manipulatives being tools that allows students to "visualize" mathematical concepts and make their own mathematical relationships. Maccini, Strickland, Gagnon, and Malmgren (2008) and Maccini and Hughes (2000) also speak about the use of CSA instructional strategy as an effective mathematical intervention for students with disabilities. Allsop (1999) and Foegen (2008) both state that the use of manipulatives and a CSA instruction is a best practice intervention for students with disabilities. The treatment group used algebra tiles throughout their introduction, instruction and practice of algebra expressions. This group was explicitly taught on how the tiles work and what they represent. Students used them on the post-test as well as during all work leading up to that test; therefore, the post-test for the treatment group included boxes for students to use algebra tiles (see Appendix C). The control group did not have access to algebra tiles and continued with drill and practice with the use of worksheets and teacher led instruction.

Procedures

Subjects of this study participated in a five-week experimental study. Participants were given a pre-test during week one and followed by the intervention in weeks two through four. During those weeks, the experimental group received lessons on algebraic expressions with the use of algebra tiles while the control group did not have access to the manipulatives. During the fifth and final week, all participants took a post-test, which was identical to the pre-test. The curriculum used for the lesson called for the chapter to take ten days. However, due to the fact that all student participants have LD, this study and lesson needed to take additional time. Van de Walle and colleagues (2016) state that students will disabilities are often left behind because they cannot keep up with the pace of a general education classroom. Students with disabilities often struggle with their memory, thus extending the time in which it may take them to learn a topic. The extra time given will help to aid students to better comprehend the topics introduced.

Data collection. Week one a pre-test was given to all student participants. Weeks two through four both groups received instruction on algebraic expressions with the experimental group receiving the intervention and the control group receiving instruction only with no manipulatives. Week five was review and ended with all student participants receiving the same pre-test in a post-test form. Data were based on the pre- and post-test scores. All questions on the instruments counted as one point and were marked either correct or incorrect. Student scores from the treatment and control group were graphed and compared.

Fidelity. Within the classrooms of both the experimental and control group there was an instructional assistant (IA) that verified that both teachers were implementing instruction as was planned. Due to the fact that there were two teachers participating in this study both met previously before the study began to verify their inter-rater reliability. They met weekly to insure the fidelity of the experiment was being kept (see Appendix D). The same was done with both IA's so that they may verify the fidelity of both groups. The researcher had 100 percent fidelity to intervention.

Ethical Considerations

This study was not notably damaging to students' physical or emotional being. The population of the participants was very sensitive to anyone knowing that they have disabilities. Confidentiality was of the utmost importance and to insure this no names were be recorded or documented throughout the study. Though information on student test scores were kept, no names were recorded as students used their identification numbers instead. Students were not linked to the classes or to a disability. There could have been issue with student frustration with the new implementation of the algebra tiles. If students were not familiar with tiles they could've become upset and shut down. This was alleviated by giving students breaks when needed and introducing the algebra tiles slowly. However, it is pertinent to the study that both teachers kept their fidelity with the study. If the intervention showed to be successful, when it was complete, it will be implemented to the control group.

Validity threats. There were two different teachers involved in this study with varying experiences of teaching mathematics to students with disabilities. This could have posed as a threat to validity as one teacher may have enhanced a student's ability to learn. To counter this issue the teachers met prior to the study and each week during the study to discuss lessons and what was acceptable and not acceptable throughout the study. The researcher provided the control group teacher with all of the materials and lesson plans used by the experimental group minus the intervention (i.e., algebra tiles). Due to the fact that the researcher was also the teacher implementing the intervention there could have been some researcher bias as the teacher was invested in the participants' scores. To combat this, the intervention and scoring process were viewed by an instructional assistant who was informed of the research study. Entire classes were used hence there should have been no sampling bias as the treatment and control group were nearly identical samples.

Data Analyses

Data were analyzed based on the scores of the pre- and post-test. Student test scores were kept and compared against each other as a whole group. Scores were looked at as progress made from pre to post-test as well as overall percentage of group scores. All data were entered into the Statistical Package for the Social Sciences (SPSS) for Windows, version 24.0.0 (SPSS, 2016). No names or identifying information were included in the stat analysis. Before analyses were conducted all data were cleaned to ensure no outliers were present (Dimitrov, 2012). Five participants were removed from the data file due to classroom changes and absenteeism, which led to the inability for them to complete the testing. After cleaning the data, the final sample size was 31 participants; 16 for the treatment group and 15 for the control group. Independent and paired sample t-tests were conducted to determine the significant difference in mathematical performance scores on *Pacemaker: Algebra 1* chapter two individual chapter test, before interpreting the analytical output, Levene's Homogeneity of Variance was examined to see if the assumption of equivalence had been violated (i.e., the variances were equal across groups), data were interpreted for the assumption of equivalence; however, if the variances were not equal across groups the corrected output was used for interpretation.

Results

Two independent samples t-tests were conducted on the whole sample ($n = 31$) for both the pre- and post assessment scores. Results for the pre-test were: Levene's Homogeneity of Variance was not violated ($p > .05$), meaning the variance between groups was not statistically different and no correction was needed, and the t-test showed non-significant differences between the mean scores on the pre-tests between the two groups $t(29) = 1.17$, $p > .05$. This shows that the starting point for both the control and treatment group were relatively the same and neither group outperformed the other (see Table 1).

Results for the post-test were: Levene's Homogeneity of Variance was not violated ($p > .05$), meaning the variance between groups was not statistically different and no correction was needed, and the t-test showed non-significant differences between the mean scores on the post-tests between the two groups $t(29) = -.721$, $p > .05$. Both groups scored relatively the same on the post-test showing that there appeared to be no benefit from the treatment (see Table 1).

Table 1

Results of Independent Samples T-Tests

	Mean	SD
Pre Test		
Treatment	1.75	1.65
Control	1.13	1.25
Post Test		
Treatment	6.06	3.32
Control	7.07	4.40

Note. SD = Standard Deviation.

After determining the differences between pre and post assessment scores between groups, two paired t-tests were run for both groups (i.e., treatment and control) to determine if participants mean scores from pre to post were significantly different within each group (see Table 2). Results for each group were as follows: treatment group, $t(15) = -5.83$, $p < .001$; control group, $t(14) = -4.42$, $p < .01$. The mean scores from pre to post-test were significantly statistically different for both groups. Additionally, the negative t-value for each group indicates an increase in scores from pre to post assessment. Both the treatment and control groups improved their scores greatly from pre- to post-test, showing that they both learned during the experiment; however, the lack of statistical difference in post-test scores demonstrate that the intervention was not any more effective than students' normal curriculum.

Table 2

Results of Paired T-Tests

	Mean	SD
Treatment Group		
Pre	1.75	1.65
Post	6.06	3.32
Control Group		
Pre	1.13	1.25
Post	7.07	4.40

Note. SD = Standard Deviation.

Discussion

The purpose of this study was to determine if the use of algebra tiles in a secondary Mild to Moderate classroom would improve comprehension of algebraic expressions with students with disabilities. This study included 19 students who received instruction (i.e., treatment) with the use of algebra tiles and 17 students who received instruction without the use of the tiles (i.e., control). My hypothesis stated that the use of algebra tiles would improve students' comprehension of algebraic expression; however, the results indicated that the use of the algebra tiles did not lead statistically significant differences. Throughout the experiment the two groups were given identical notes and problems, however the treatment group was required to use algebra tiles to show and complete their work during the study. The results based on both groups' test scores showed that the control group displayed slightly higher growth from pre to post-test than the treatment but not enough to come to any definitive conclusions. Therefore, the initial hypothesis was partially accepted because the treatment group's mean score increased, but not enough for a statistically meaningful difference compared to the control group on the post test.

During the pre-test phase, the standard deviation (SD) of the scores of the treatment group was slightly higher than that of the control group. However following the post-test the SD of the treatment group was lower than that of the control group. The SD measures how variable the data are around the mean, and the lower the SD the closer the scores are centered around the mean. The treatment group had a lower SD on the post-test indicating that there were less outliers and a larger amount of the scores were closer to the mean. Though neither of the groups' scores were significantly different, this did show that the treatment group was learning more consistently as a whole group.

This study was unable to achieve the same results as in studies by Maccini and Ruhl (2000) and Maccini and Hughes (2000) in which algebra tiles showed a positive growth with student comprehension with algebraic topics. However this study also did not show a decline of student skill with the use of manipulatives as in a study by McClung (1998). This study did show that there was no significant statistical benefit from the use of algebra tiles. This is consistent with the findings in a study by Dyer (1996) in which students taught with tiles scored the same on a measure of retention as students taught with just the textbook. It appears that there is no true consensus on the effects of algebra tiles and student comprehension of algebraic concepts. This leads to needing more studies to be conducted with larger population of students.

Limitations & Future Studies

Some limitations to the study could have been the sample size, which was relatively small. A larger sample size would allow for the results to be more reliable and easier to generalize across larger populations. Another limitation to the study was the added layer of having students draw out the tiles during the post-test (see Appendix C). The treatment group was asked to show their work on the post-test by drawing out the tiles on their test sheet. This could have posed a problem for the treatment group as it added an extra component to the solving the problem. It should also be mentioned that the text and test used for the study was not specifically designed for the use of algebra tiles. Therefore, certain problems required a large number of tiles that made it difficult for students to apply the tiles to solve the problem. The use of a test that is structured for algebra tiles ensures that students are being measured on their ability to comprehend algebraic expressions rather than their ability to organize tiles. Lastly, the sample chosen was a convenience sample which limits the ability of the study to be generalized to a larger population and there can be some underlying bias in the study. In future studies it would be best use a larger and more random sample size.

Future studies should follow with a CSA model such as those used in studies by Maccini and Ruhl (2000) and Strickland and Maccini (2012). As this strategy slowly fades out, the use of the algebra tiles or manipulatives as students gain mastery of the concept. As the use of manipulatives will not always be available it is best that students eventually become able to solve the problems just with the use of abstract numbers and symbols. Subsequent studies should also have larger sample sizes that reflect a larger portion of the general population. This will aid in the studies overall ability to be generalized as well as its validity. Finally, future studies should utilize pre- and post-tests that are designed for the use of algebra tiles. If a test is not compatible with the use of algebra tiles, it can put the user of the tiles at a disadvantage due to the number of tiles some problems may require. Future studies need to continue to be conducted to ensure that students with disabilities are better able to access the general education curriculum.

Summary

It is important for educators to be willing to try new and different methods of teaching in order to benefit their student population. Though this study did not obtain the results the researcher hypothesized, it did show that students with disabilities are able to learn abstract algebraic concepts using both traditional instruction and algebra tiles. This is especially encouraging because of the struggle students with disabilities have with algebra (Foegen, 2008). Thus future studies should continue with the idea that students with disabilities are able to learn algebraic concepts and look for equitable ways to teach this population of students.

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Appendix A

Pre-Test: Control and Treatment Groups

Name:_________________ Date:_______

Simplify

1. $10 - 4 \div 2$
2. $3 \cdot 4 + 7$
3. $8 \div 2 - 6 \cdot 0$
4. $12 \div (3 + 3) \cdot 6$
5. $20 - 2(16 - 4)$

Evaluate each variable expression

6. $(15 \div a)a$; when a is 5
7. $s + 2s$; when s is 4
8. $-5s$; when s is 7
9. $6 + c$; when c is 13

Simplify each expression

10. $4x + x + 3$
11. $3w + 4x - 7x - w$
12. $1 + 2b + 7b$
13. $4(y + 3)$
14. $-(a + 9)$
15. $6(x - 8)$
16. $3x + 5(4 + x)$
17. $12 - (c + 8)$

Appendix B

Post-test: Control

Simplify

1. $12 \div (3+3) \cdot 6$
2. $10 - 4 \div 2$
3. $20 - 2(16 - 4)$
4. $8 \div 2 - 6 \cdot 0$
5. $3 \cdot 4 + 7$

Evaluate each variable expression

6. $6 + c$; when c is 13
7. $-5s$; when s is 7
8. $(15 \div a)a$; when a is 5
9. $s + 2s$; when s is 4

Simplify each expression

10. $12 - (c + 8)$
11. $3x + 5(4 + x)$
12. $-(a + 9)$
13. $6(x - 8)$
14. $4(y + 3)$
15. $1 + 2b + 7b$
16. $3w + 4x - 7x - w$
17. $4x + x + 3$

Appendix C

Post-Test: Treatment

Simplify

1. $12 \div (3 + 3) \cdot 6$

2. $10 - 4 \div 2$

3. $20 - 2(16 - 4)$

4. $8 \div 2 - 6 \cdot 0$

5. $3 \cdot 4 + 7$

Evaluate and model each expression

6. 6 + c ; when c is 13

Model of the Expression	**Expression with rectangles replaced**	**Simplified Answer**

7. ‑ 5s ; when s is 7

Model of the Expression	**Expression with rectangles replaced**	**Simplified Answer**

8. (15 ÷ a)a ; when a is 5

Model of the Expression	**Expression with rectangles replaced**	**Simplified Answer**

9. s + 2s ; when s is 4

Model of the Expression	**Expression with rectangles replaced**	**Simplified Answer**

Simplify each expression

10. 12 – (c + 8) Simplified Answer:________________

11. 3x + 5(4 + x) Simplified Answer:________________________

12. $-(a + 9)$ Simplified Answer:______________________

13. $6(x - 8)$ Simplified Answer:______________________

14. $4(y + 3)$ Simplified Answer:_____________________

15. $1 + 2b + 7b$ Simplified Answer:______________________

16. $3w + 4x - 7x - w$ Simplified Answer:______________________

17. $4x + x + 3$ Simplified Answer:______________________

Appendix D

Fidelity Checklist

Fidelity Observation Chart

Date	Treatment/Control	Signature
2/21/17	T	[illegible]
2/23/17	T	[illegible]
2/27/17	C	[illegible]
3/01/17	T	[illegible]
3/02/17	C	[illegible]
3/06/17	T	[illegible]
3/08/17	C	[illegible]
3/09/17	T	[illegible]
3/13/17	T	[illegible]
3/15/17	C	[illegible]
3/16/17	C	[illegible]
3/20/17	T	[illegible]
3/22/17	C	[illegible]
3/23/17	T	[illegible]
3/27/17	T	[illegible]
3/29/17	C	[illegible]