Spring 2019

Using Math Manipulatives with Students of Color

Karin Zandakis
California State University, Monterey Bay

Using Math Manipulatives with Students of Color

Karin Zandakis

Thesis Submitted in Partial Fulfillment of the Requirements for the
Degree of Master of Arts in Education

California State University, Monterey Bay

May 2019

Using Math Manipulatives with Students of Color

Karin Zandakis

APPROVED BY THE GRADUATE ADVISORY COMMITTEE

____

Kerrie Chitwood, Ph.D.
Advisor and Program Coordinator, Master of Arts in Education

____

Dennis Kombe, Ph.D.
Advisor, Master of Arts in Education

Approved by the Dean of Graduate Studies 05/21/2019

Kris Roney, Ph.D. Associate Vice President
Academic Programs and Dean of Undergraduate & Graduate Studies

Abstract

An opportunity gap, or achievement gap, exists between students of color and white students in the American public school system and this gap can be seen in math in particular (Coleman, 2018). The present body of literature suggests that using math manipulatives, or concrete objects, can help students learn abstract math concepts and mathematical reasoning. However, current research does not specify whether the use of math manipulatives can be used to help students of color improve their math scores. This study used a pre-test and post-test quantitative quasi-experimental design to look at the use of math manipulatives in a fourth-grade classroom to see if students of color would improve on a math assessment after using math manipulatives every day for five weeks. Study participants were 43 fourth grade students at an elementary school on the central coast of California. The treatment group was comprised of 22 students and the control group had 21 students. Analysis of the independent and paired t-test showed an increase in the mean scores for the treatment group in the posttest compared to their mean scores on the pretest. These numbers were statistically significant, which shows that the intervention was effective for helping students of color improve in math. Further research is needed to continue investigating the effects of math manipulatives on closing the opportunity gap.

Keywords: opportunity gap, achievement gap, math manipulatives, students of color, school segregation, white privilege

Acknowledgements

To my professors, Kerrie Chitwood, Dennis Kombe and Erin Ramirez for editing my thesis and providing valuable feedback. Thanks to my partner teacher, Hannah Cobb, who taught the control group. Finally, I would like to thank my wife, Corrina, who read every single draft of my thesis and believed in me even when I wavered.

TABLE OF CONTENTS

Using Math Manipulatives with Students of Color

Literature Review

Students of color underperform on math tests and assessments compared to their white peers; this is referred to as the *achievement gap* or the *opportunity gap* (Anderson, Medrich, & Fowler, 2007; Kotok, 2017; Miretzky, Chennault, & Fraynd, 2016). The National Center for Education Statistics (2010) notes that an educational achievement gap exists when one group of students is performing academically better than another group and the difference between the average scores in both groups is statistically significant. Furthermore, opportunity gap denotes the historical significance of racial disparities in terms of equal access to opportunity (Pitre, 2014). For the purposes of this literature review, opportunity gap will be used to reference the difference in math test scores between white students and students of color. The opportunity gap in mathematics is specifically important because it is crucial for students of color have access to equitable math education to prepare for jobs in science, technology, engineering, and mathematics (Kotok, 2017).

When white students perform better than students of color on math tests, it is a reflection of the resources, cultural background, and opportunities white students have rather than something intrinsic within the individual student (Better, 2008). McIntosh (1988) describes this as white privilege – an "invisible knapsack" of unearned advantages white people always have, even if they are unable to name or recognize this privilege. While social, cultural, and economic advantages are given freely to those with white privilege, they are systematically denied to people of color (Ladson-Billings & Tate, 1995). This advantage leads to an opportunity gap that is present as early as kindergarten.

Black and Latino students in kindergarten have lower test scores than their white peers

(Reardon & Galindo, 2011). As students progress through school, this gap continues to grow (Kotok, 2017; Reardon & Galindo, 2011). Studies show the opportunity gap can lead to tracking into lower-level math courses in secondary education, lower college admissions, increased dropout rates, underemployment, and lower wages for people of color (Achieve, 2013; Johnson, 2006; Kotok, 2017). One of the reasons why the opportunity gap continues to grow for students of color is because the tests used to assess student achievement are biased, thereby giving white students an unfair advantage (Better, 2008; "Courageous Conversations," 2018).

Whiteness as the Norm

One reason why the opportunity gap exists is because tests favor students who have been brought up in white culture and have white privilege (Better, 2008). In America, everyone must navigate whiteness because white culture dominates other cultures and is seen as the norm (Johnson, 2006; Rothenberg, 2010). This dominance of one group over other cultures is known as hegemony and it affects all areas of life, including schools. Students, including students of color, are rewarded for performing whiteness and conforming to white norms (Ladson-Billings & Tate, 1995). Educators, even educators of color, teach white culture and instruct students how to navigate white culture ("Courageous Conversations," 2018). Consciously or not, test writers assume all students have the experiences, vocabulary, and knowledge that predominate white culture ("Courageous Conversations," 2018). Due to de facto segregation, students of color have different lived experiences, vocabulary, and reference points which, while valid, do not prepare students to take tests based on white culture (Better, 2008).

Segregation in Schools

Brown v. Board of Education (1954) declared segregated schools are unconstitutional; however, the American public school system is more segregated now than it was before integration, and many inequities prevail (Better, 2008; Ladson-Billings & Tate, 1995; Orfield, Frankenberg, & Siegel-Hawley, 2016). California currently has one of the most segregated public school systems due to the rising Latino student population (Orfield et al., 2016). Researchers found Latino students in California have less interaction with white students than students of color in any other state; 90% of students of color in California attend schools that serve a majority of students of color (Orfield et al., 2016). Nationally, black students attend more racially and economically segregated schools than any other racial group (Kotok, 2017).

Researchers found black and Latino students are segregated into schools that are typically low performing, especially compared to schools that serve a majority of white students (Kotok, 2017; Orfield et al., 2016). Low-income schools and schools that serve a high percentage of students of color typically have inadequate facilities, less qualified teachers, face more behavioral problems, and offer less high-level course choices at the middle and high school levels (Akiba, LeTendre, & Scribner, 2007; Kotok, 2017; Ladson-Billings & Tate, 1995; Orfield et al., 2016; Rothenberg, 2010). Kotok (2017) also noted high-achieving students of color who attend low-performing schools may still perform poorly on standardized math tests due to inequities caused by school segregation. Additionally, students of color at integrated schools often face alienation from peers of the same race if they choose to take advanced math classes, but students of color at segregated schools do not face the same stigma because tracking is not race related at racially homogenous schools (Kotok, 2017). However, students of color who attend segregated schools are consistently denied the *opportunity to* *learn* because there is less access to educational materials and resources (Kotok, 2017; Ladson-Billings & Tate, 1995).

Due to economic and social inequalities, people of color are segregated into communities that do not have access to the best job opportunities, schools and healthcare services (Johnson, 2006; Kotok, 2017). Public school budgets are reliant on property taxes, which allow people living in higher income areas to have access to better schools than those living in low-income areas (Ladson-Billings & Tate, 1995; Rothenberg, 2010). Although low-income schools receive Title I funds, schools that predominantly serve students of color are still underfunded compared to their needs (McCarthy, Eckes, & Decker, 2019; Rothenberg, 2010). For example, even if a low-income school receives Title I funds, they may still have inadequate facilities, outdated textbooks, and undertrained teachers. Schools receive the majority of their funding from local sources and Tittle 1 funding is not intended to equalize funding within states (Rothenberg, 2010). Even though the Supreme Court ruled against legal segregation in *Brown v. Board of Education* (1954), the court made no mention of de facto segregation. By staying silent on de facto segregation, the Supreme Court allowed housing and school segregation to flourish, resulting in schools and communities that are more segregated now than they were 60 years ago (Orfield et al., 2016; Rothenberg, 2010).

All of these factors contribute to the opportunity gap and students of color obtaining lower test scores on math assessments than white students (Johnson, 2006; Kotok, 2017). Students of color face centuries of racial segregation and unequal access to quality education that the American public school system continues to perpetuate to this day (Rothenberg, 2010). Educators and schools need to do everything in their power to lessen the effects of segregation and institutionalized racism that dominate the American public school system.

One of the many difficulties students of color are combatting is the opportunity gap in the subject of mathematics (Coleman, 2018).

The Opportunity Gap in Math

According to the National Center for Education Statistics, black and Latino students in California scored 29 and 28 points lower than white students on the National Assessment of Education Progress (Hemphill, Vanneman, & Rahman, 2011; Vanneman, 2009). In districts along the central coast, approximately 54% of Latino students did not meet the math standards on the California Assessment of Student Performance and Progress (CAASPP), compared to 50% of Asian students, 33% of bi/multiracial students and only 21% of white students who did not meet the standards on the CAASPP (Coleman, 2018). Educators in the region are aware of an opportunity gap between the students of color and their white peers; however, at this time there is not an articulated plan to reduce the opportunity gap. At the school where the present study was conducted, an opportunity gap currently exists between students of color and white students in all areas, but the gap is especially prevalent in the field of mathematics as evidenced by the CAASPP scores (Coleman, 2018). Although the opportunity gap can be seen in CAASPP scores in students as young as 3rd grade (Coleman, 2018), the opportunity gap only continues to widen as students enter middle and high school (Johnson, 2006; Kotok, 2017).

By the time students of color are in secondary education, they have already been denied the opportunity to learn and are being tracked into lower-performing schools and lower math courses, such as general math compared to geometry (Achieve, 2013; Kotok, 2017; Ladson-Billings & Tate, 1995). Kotok (2017) also notes that students of color at integrated schools are more likely than white students to be tracked into lower-level math courses, even when they score in top percentiles on standardized math tests. Although black students show the most interest in pursuing science, technology, engineering or math (STEM) degrees in college, they are the least likely to be mathematically prepared for these courses (Achieve, 2013; Kotok, 2017). Achieve Inc. (2013) reported less than a third of schools that serve a majority of students of color offer advanced math classes such as Calculus, compared to half of all schools nationally. Higher-level math courses are a barrier to entry for students of color because taking advanced math courses is directly linked to post-secondary advantages such as college graduation and future post-graduate earnings after students finish school (Achieve, 2013; Kotok, 2017).

Although it is clear an opportunity gap exists between white students and students of color in mathematics, it is unclear is how to reduce the gap. School districts and educators have an obligation to give all students the opportunity to learn. Several researchers have studied best practices in mathematical instruction and how to reduce the opportunity gap between students of color and white students in math (Achieve, 2013; Kotok, 2017).

Interventions and Suggestions to Close the Opportunity Gap

Many researchers have identified the opportunity gap as a problem and have studied different school-based interventions to try and reduce the opportunity gap. For example, Pitre (2014) suggests teachers use meaningful learning experiences, academic rigor, cultural connections, and a profound belief in students' capabilities to reduce the opportunity gap. Teachers' expectations of students' abilities have a huge impact on student success (Kotok, 2017; Rothenberg, 2010). For the most part, students will live up to the expectations set by their teachers; generally, teachers have higher expectations for white students and lower expectations for students of color (Rothenberg, 2010). Low teacher expectations in math can lead to low self-efficacy, or self-confidence, for students of color (Kotok, 2017). If teachers do not expect students of color to succeed, they will not be worried when students of color perform poorly, and both teachers and students will accept low grades (Rothenberg, 2010). Low teacher expectations coupled with low student self-efficacy in math discourages students of color from taking advanced math classes or pursuing STEM degrees in college (Kotok, 2017). This becomes a vicious and self-reinforcing cycle that perpetuates the opportunity gap in mathematics (Rothenberg, 2010).

Math teachers often feel pressed for time and turn to direct instruction in order to maximize their instructional minutes (Joyce, Weil, & Calhoun, 2009). According to Joyce and colleagues (2009), direct instruction is a model of teaching that is often favored by teachers because it prioritizes academic tasks and allows the teacher greater control over the class. However, direct instruction is a teacher-centered technique and is not necessarily the best strategy for teaching diverse learners, including students of color (Carbonneau, Selig, & Marley, 2013). If direct instruction is the only method of delivery used in a classroom, students with different learning styles can fall behind (Liggett, 2017). As students of color fall further behind in math, their enthusiasm for the subject dwindles and they are less likely to pursue advanced math in secondary education and beyond (Kotok, 2017, Liggett, 2017). In order to ensure students of color stay engaged and continue to enjoy math, teachers should use engaging models of teaching as an alternative to direct instruction whenever possible (Joyce et al., 2009; Pitre, 2014).

One of Pitre's (2014) suggestions to close the opportunity gap is for teachers to give students meaningful learning experiences, multiple opportunities to practice, and real-world relatability. Due to the fact that developing math skills can be challenging for young children, students need critical reasoning skills in the field of mathematics in order to be able to function in the world (Liggett, 2017). Teachers need to provide opportunities for students to experience math through a concrete-to-abstract sequence of instruction as a way to ensure that they have a thorough understanding of the math concepts that they are learning (Ladson-Billings & Tate, 1995). Researchers have found students are able to develop their mathematical reasoning skills through the use of math manipulatives (Carbonneau et al., 2013; Furner, Yahya, & Duffy, 2005). Math manipulatives are any concrete, physical objects used by teachers for math instruction with the purpose of helping students understand abstract math (Liggett, 2017). Math manipulatives can include pattern blocks, fraction strips, fraction circles or any other physical object that could be used to teach math.

Numerous studies have shown use of concrete manipulatives help students to have a better understanding of abstract math than students who receive direct instruction on mathematical concepts (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005). Liggett (2017) found once students were introduced to math manipulatives, they were able to self-select objects in their learning environment to help increase their own understanding of math. Understandings gained through the use of math manipulatives might have otherwise gone unnoticed if the students were only taught abstract math (Liggett, 2017). Furthermore, Fujimura (2001) found students who are given opportunities to practice using concrete math manipulatives made bigger gains than students who were not provided with manipulatives. Additionally, students of all ages, varying ability, and levels of understanding were able to engage meaningfully in mathematics through the use of hands-on math manipulatives (Liggett, 2017). Furner and colleagues (2005) found diverse learners (i.e., students of color, students with disabilities, and English language learners) benefitted from using concrete manipulatives during math instruction before learning abstract math. Through the use of math manipulatives, students were able to reach a level of comprehension that was previously inaccessible (Liggett, 2017). In addition, Liggett (2017) found students with access to math manipulatives were able to apply their learning from one concept to another mathematical topic. Many researchers have found the use of math manipulatives can assist students in understanding abstract math in a concrete way, and researchers have also found students of color benefit from engaging and hands-on mathematical learning (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017; Pitre, 2014).

While there are studies indicating meaningful learning experiences may help close the opportunity gap and improve students' test scores, and using math manipulatives helps students to improve in math, a gap exists in the research tying these two ideas together (Fujimura, 2001; Furner et al., 2005; Liggett, 2017; Pitre, 2014). Although researchers and educators agree there is an opportunity gap between students of color and white students in the field of mathematics, the literature does not agree on a solution. Researchers have studied possible interventions to reduce the opportunity gap; however, there is no conclusive solution. Researchers agree using hands-on strategies during math instruction helps students understand abstract math through the use of concrete math manipulatives (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017). Although researchers agree an opportunity gap does exist between students of color and white students in mathematics and math manipulatives help students understand complex math, there is no research about the use of math manipulatives to specifically help students of color improve their scores on math assessments.

Method

The purpose of this study was to use math manipulatives as an intervention strategy in a classroom of twenty-two 4th grade students to help students of color improve their math scores. Math manipulatives are any concrete object that the teacher uses in the classroom for the purposes of helping students understand abstract mathematical concepts (Liggett, 2017). Some examples of math manipulatives that help with understanding fractions include: pattern blocks, unit cubes, fraction strips and fraction circles. Specifically, this study looked at using math manipulatives during math instruction to give students of color more meaningful learning experiences to determine if their test scores improved compared to students of color that did not have access to manipulatives. The researcher chose to use math manipulatives as the intervention strategy because providing meaningful learning experiences through the use of concrete manipulatives has been shown to help a variety of students succeed in math (Carbonneau, Selig, & Marley, 2013; Furner, Yahya, & Duffy, 2005; "Grade 4 Mathematics," 2014).

Research Question

The research question for this study was: Does using math manipulatives during math instruction help 4th grade students of color at an elementary school in Central California improve their math assessment scores as evidenced by their performance on the Eureka Math Mid-Module 5 Assessment?

Hypothesis

The researcher hypothesized that using concrete math manipulatives would help 4th grade students of color improve their math assessment scores compared to students of color that did not receive the intervention as evidenced on the Eureka Math Mid-Module 5 Assessment (Carbonneau et al., 2013; Furner et al., 2005; Pitre, 2014).

Research Design

The present study was a quantitative, quasi-experimental study with nonequivalent groups, pretest-posttest design. There were two groups: a treatment and a control group. Both groups took the pretest and the posttest, but only the treatment group received concrete math manipulatives as an intervention. This study took approximately 25 school days, or five weeks, to complete.

Independent variable. The independent variable in this study was the use of concrete math manipulatives. Math manipulatives were defined as any physical object that might help the students understand abstract concepts (e.g., fraction towers, pattern blocks, fraction strips and fraction circles; Furner et al., 2005; Liggett, 2017).

Dependent variable. The dependent variable in this study was student scores on the Eureka Math Mid-Module 5 Assessment which assessed student knowledge of fractions ("Grade 4 Mathematics," 2014). Conceptually, the Mid-Module assessment measured what students learned during the course of one half of a math module. Once the assessments were graded, student scores were entered into the school district's data base and the Statistical Package for the Social Sciences ® (SPSS®) and were analyzed by demographic information (SPSS®, 2016).

Setting & Participants

The study took place at an elementary school in Central California. According to demographic information for the school where the study took place, 19% of the students are white, 15.7% of the students are bi or multiracial, 2.3% are Native Hawaiian or Pacific Islander, 46.2% are Hispanic or Latino, 5.6% are Filipino, 4.3% are Black or African American, 6.3% are Asian, and 0.5% are Native American (Education Data Partnership, 2018). This study used a purposeful convenience sample; students of color in the researcher's class made up the treatment group and students of color in another 4th grade classroom comprised the control group. The treatment group and the control group were roughly equivalent, although there were small differences in the demographics and the scores on the first math assessment of the year. There were 22 students in the treatment group and 21 students in the control group, yielding a total of 43 participants.

Treatment group. There were 22 students in the treatment group. The treatment group was 32% Hispanic/Latino, 9% black/African American, 46% bi or multiracial, 9% Asian, and 4% Pacific Islander. 41% of the students were female and 59% of the students were male. On the first math assessment this year, 15.4% of students exceeded the standards, 7.7% met the standards, 42.3% nearly met the standards, and 34.6% were below the standards. In the treatment group, 20 students did not meet the standards on this math assessment and 18 of those students were students of color compared to only two white students.

Control group. There were 21 students in the control group. The control group was 57% Hispanic/Latino, 5% black/African American, 28% bi or multiracial, and 10% Asian. 47% of the students were female and 53% of the students were male. On the first math assessment, 12.5% of the students exceeded the standards, 33% met the standard, 20.8% nearly met the standards, and 33.3% were below the standards. In the control group, 13 students did not meet the standards on this math assessment and 12 of those students were students of color compared to only one white student.

Measures

The researcher used the Eureka Math Mid-Module 5 Assessment from the 4th grade Eureka Math curriculum in this study ("Grade 4 Mathematics," 2014). This is an assessment that is included in the Eureka Math curriculum (see Appendix A) and is evaluated every year by grade level representatives and Teachers on Special Assignment (TOSAs) in the school district. The Eureka Math Mid-Module 5 Assessment has six questions and is made up of both word and computational problems. The Eureka Math Mid-Module 5 Assessment is five pages long and took both classes around two hours to finish. The assessment is a paper/pencil test that is hand-scored by the grade level team and the academic coach using a rubric and an answer key (see Appendix A and B).

Validity. This assessment has a high level of validity as each of the questions are aligned with the standard being measured (see Appendix B). Since each question is aligned with a specific standard, the Eureka Math Mid-Module 5 Assessment has a high level of construct validity and the assessment measures what it claims to measure ("Grade 4 Mathematics," 2014). This assessment is also examined by a group of teachers and TOSAs every year to ensure face validity.

Reliability. The Eureka Math Mid-Module 5 Assessment was determined to be reliable because multiple questions addressed each standard on the test, which shows internal consistency ("Grade 4 Mathematics," 2014). Additionally, the curriculum provides teachers with an answer key and a rubric to grade the assessments (see Appendix A and B). When grading these assessments, the researcher met with the grade level team, including the academic coach and established inter-rater reliability. The researcher, the teacher for the control group and the academic coach each graded 33.33% of the pre and post assessments for the control and treatment groups to ensure 100% inter-rater reliability (McMillan, 2016). Therefore, the Eureka Math Mid-Module 5 Assessment can be used in this study without hesitation.

Intervention

The intervention group in this study used math manipulatives daily during math instruction in order to learn abstract math in a concrete way. Math manipulatives are physical objects that students can use to gain a concrete understanding of abstract mathematical concepts (Furner et al., 2005; Liggett, 2017). Examples of math manipulatives used are fraction strips, fraction circles, playdough, unit cubes, and pattern blocks. Fujimura (2001) found students who are given opportunities for hands-on learning using manipulatives made greater gains in mathematics than students who only received direct instruction. The teacher demonstrated different ways to use the math manipulatives on the document camera and then distributed the manipulatives to the students. Students had one to two minutes to explore how to use the manipulatives on their own before the teacher brought the class back together. The teacher did several example problems on the document camera with the manipulatives using the gradual release of responsibility model. After completing the whole group part of the lesson, students went to their small group stations and had additional opportunities to use math manipulatives while they completed their independent work. While the students worked at their stations, the teacher worked with one small group at a time and helped students solve fraction problems with the use of math manipulatives. The researcher used manipulatives during math instruction with the treatment group while the teacher for the control group followed the scripted lesson plan in the Eureka Math curriculum and did not use math manipulatives ("Grade 4 Mathematics," 2014).

Procedures

This study began when both the treatment group and the control group started the second half of Module 5 of the fourth grade Eureka Math curriculum. Both classes took a pretest (i.e., the Eureka Math Mid-Module 5 Assessment from the curriculum) on the topic before instruction was given to measure prior knowledge. The teacher for the control group taught the scripted math curriculum from the Eureka Math Teacher's Edition without the aid of math manipulatives. The researcher taught the treatment group the same curriculum, but supplemented the curriculum using district provided math manipulatives. The second half of the Eureka Math Module 5 curriculum included 20 lessons and the average lesson took one day to complete, so the researcher used math manipulatives with the treatment group for 20 instructional days. Including days for the pretest, review and the posttest, this study took 25 days, or roughly five weeks, to complete. The academic coach visited both the control group and the treatment group twice to ensure fidelity (see Appendix C).

At the end of the module, both the control group and the treatment group took the same Eureka Math Mid-Module 5 Assessment on the same day and with the same time constraints. Once both classes finished taking the assessment, the grade level team and the academic coach graded the tests together using the answer key and rubric (see Appendix A and B) to ensure reliability and validity. The researcher measured the opportunity gap by inputting student scores into the school district's online data system and SPSS® (SPSS®, 2016), then analyzed the scores based on racial demographic data. If the researcher's hypothesis was upheld, then the students of color in the intervention group would make more academic growth than the students of color in the control group.

Data collection. Data collection occurred during the pretest and the posttest. The Eureka Math Mid-Module 5 Assessment was used for both the pretest and the posttest. Both the treatment group and the control group took the Eureka Math Mid-Module 5 Assessments and the researcher graded the assessments with the grade level team and the academic coach using an answer key and a rubric to ensure reliability (see Appendix A and B). Then the researcher entered the grades into the district database and SPSS® (SPSS®, 2016) and the results were broken down by race. No other data was collected during this study.

Fidelity. Fidelity was monitored by the academic coach who observed both the treatment group and the control group to check that the treatment group received the intervention and that the control group did not. The academic coach was looking at both groups using the fidelity checklist in Appendix C to ensure that the control group was following the scripted Eureka Math curriculum and that the treatment group was using manipulatives daily. The academic coach observed each group twice, for a total of four days, which was 20% of the intervention period (see Appendix C). The treatment group received the intervention 100% of the time and the control group did not use the intervention at all during this time period. By ensuring fidelity to the intervention, the researcher was able to determine whether or not the intervention was successful for the treatment group.

Ethical Considerations

One of the ethical considerations in this study was student access to manipulatives based on which group they were in (i.e., control group or treatment group). This was an ethical consideration because multiple studies show concrete manipulatives help students understand abstract math (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005). Once the experiment ended, the control group was given access to math manipulatives for the remainder of the school year.

Other ethical considerations were student confidentiality and informed consent. The researcher had to obtain informed consent from the students and the students' parents. Additionally, the researcher could not mention any identifying details about the school where the study was taking place or the information about individual students to protect student confidentiality and anonymity.

One final ethical consideration was the amount of time that students had to spend taking the pretest and the posttest. Students in both the control group and the treatment group took multiple days to complete the pretest and the posttest. This is an ethical consideration because students had to spend considerable time being assessed for this study. In addition, the posttest was administered the week before standardized state testing began and some students may have experienced testing fatigue.

Validity threats. One threat to validity was extraneous variables. One extraneous variable were the teachers for the control and intervention groups. Both teachers have different teaching styles and are not always in sync with each other in terms of the pacing guide or instructional methods. Both teachers experienced difficulty implementing the intervention (or not implementing the intervention, in the case of the control group) with fidelity. The teacher for the intervention group used both class sets of manipulatives to ensure that the intervention group always had access to manipulatives and that the control group did not have access to manipulatives. The academic coach visited both classrooms to ensure fidelity of intervention. Another tool both teachers took advantage of was the district provided SWIVL. The SWIVL is an automated video recorder which follows the teacher as they move through the classroom and the researcher and the observer were able to view these recordings to make sure the treatment group received the intervention and the control group did not. While the teacher was using the SWIVL, they wore a device that connects with the camera so only the teacher is filmed instead of the students. Using a device like the SWIVL helped both teachers be mindful of their teaching and their fidelity to the intervention.

Another threat to validity was the researcher's own bias. The researcher believed the hypothesis was correct, which could have resulted in the temptation to give students of color additional attention and support in the classroom in order to prove the hypothesis true. The researcher controlled for their own bias by setting up heterogeneous math groups and met with each group for the same amount of time each week. In this way, all students received an equal amount of small group time with the researcher.

Proposed Data Analysis

All data was entered into the SPSS® for Windows, version 24.0.0 (SPSS®, 2016). No names or identifying information were included in the data analysis. Before analyses were conducted, all data was cleaned to ensure no outliers were present (Dimitrov, 2012). After cleaning the data, independent samples t-tests (control and treatment groups) and dependent samples t-tests (pretest and posttest) were conducted to determine the significant difference in the opportunity gap between the two means scores on the Eureka Math Mid-Module 5 Assessment. Further, before interpreting the analytical output, Levene's Homogeneity of Variance was examined to see if the assumption of equivalence had been violated (Levene, 1960). If Levene's Homogeneity of Variance was not violated (i.e., the variances were equal across groups), data was interpreted for the assumption of equivalence; however, if the variances are not equal across groups the corrected output was used for interpretation.

Results

Two independent samples t-test were conducted on the whole sample ($n = 43$) for both the pre and post assessment scores. Results for the pre-test were: Levene's Homogeneity of Variance was not violated ($p > .05$), meaning the variance between groups was not statistically different and no correction was needed and the t-test showed non-significant differences between the mean scores on the pre-tests between the two groups t (41) = 1.092, $p > .05$. The treatment group has a mean of 14.77 the control group had a mean of 13.05 (see Table 1). Results for the post-test were: Levene's Homogeneity of Variance was not violated ($p > .05$), meaning the variance between groups was not statistically different and no correction was need and the t-test showed non-significant differences between the mean scores on the post-tests between the two groups t (41) = 2.769, $p > .05$. The differences between the two groups are not statistically significant so the groups are still similar. After the intervention, the groups were still comparable (see Table 1).

Table 1

Results of Independent Samples T-Tests

	Mean	SD
Pre Test		
Treatment	14.77	4.918
Control	13.05	5.436
Post Test		
Treatment	16.59	4.382
Control	12.57	5.124

Note. SD = Standard Deviation.

After determining the differences between pre and post assessment scores between groups, two paired t-tests were run for both groups (i.e., treatment and control) to determine if participants' mean scores from pre to post were significantly different within each group (See Table 2). Results for each group were as follows: treatment group, t (21) = -3.922, $p<.001$; control group, t (20) = .755, $p>.05$. The control group was consistent; the mean score decreased by .476 compared to the treatment group whose mean score increased by 1.818 points. The treatment group had a statistically significant change in their scores.

Table 2

Results of Paired T-Tests

	Mean	SD
Treatment Group*		
Pre	14.77	4.818
Post	16.59	4.382
Control Group		
Pre	13.05	5.436
Post	12.57	5.124

Note. SD = Standard Deviation. * = $p < .001$

Discussion

Multiple studies have shown that math manipulatives help students understand abstract math in a concrete way (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017). Additionally, several studies found that students of color consistently underperform in mathematics compared to white students (Coleman, 2018; Hemphill, Vanneman, & Rahman, 2011; Vanneman, 2009). Previous research indicated that the use of hands-on strategies helped students of color to perform better on math assessments (Liggett, 2017). Additionally, previous research on instructional tools for teaching math showed that math manipulatives are a useful tool for helping all students in mathematics (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017; Pitre, 2014). This study is unique because although researchers have shown that math manipulatives help students master mathematical content and studies have shown that students of color are falling behind in math, the current body of research does not contain information about the use of math manipulatives to specifically help students of color improve math assessment scores (Johnson, 2006; Kotok, 2017; Liggett, 2017).

The purpose of this study was to determine if the use of math manipulatives during math instruction was effective in helping students of color perform better on a math assessment and achieve growth in mathematics. The researcher used the Eureka Math Mid-Module 5 Assessment as both a pretest and a posttest to measure the growth that students of color made over the course of Eureka Math Module 5 (“Grade 4 Mathematics,” 2014). The intervention was the daily use of math manipulatives with the treatment group over the course of five weeks of lessons on fractions while the control group followed the scripted curriculum without the use of manipulatives. Data analysis showed that the treatment group’s mean scores increased from the pretest to the posttest (see Table 2). The researcher expected the treatment group’s mean score to increase since the students had access to math manipulatives every day which other researchers have shown to be effective in teaching math (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017). While the treatment group’s mean score on the posttest was higher than the control group’s mean posttest score, these differences were not statistically significant (see Table 1).

This study aligns with past research on the effectiveness of using manipulatives to help students achieve more growth in mathematics (Carbonneau et al., 2013; Fujimura, 2001; Furner et al., 2005; Liggett, 2017). Fujimura (2001) found that students that had access to math manipulatives when studying abstract math made bigger gains than students that did not have access to math manipulatives. This study also supports Pitre’s (2014) findings that students of color benefit from hands-on learning and meaningful learning experiences as strategies to close the opportunity gap. Specifically, past research found that the use of concrete math manipulatives helps students to cement their mathematical learning, especially if they are learning abstract math (Carbonneau et al., 2013; Furner, Yahya, & Duffy, 2005). Liggett (2017) also found that students and teachers can use any physical object as a math manipulative and that students from a range of abilities can engage with math if they have access to math manipulatives. Although most of the research surrounding the use of math manipulatives did not focus on race, Fujimura (2001) used math manipulatives in a Japanese school with all Japanese students and found that math manipulatives were effective with helping students make growth in math. Researchers point to the fact that teachers need to have concrete activities for students to engage in to understand abstract math (Carbonneau et al., 2013; Fujimura, 2001; Furner, Yahya, & Duffy, 2005; Liggett, 2017).

Limitations and Future Directions

One of the limitations of this study is that part of the intervention period coincided with a two-week spring break. The researcher implemented the intervention for three weeks, then the students went on spring break for two weeks, and after the break the researcher implemented the intervention for another two weeks before administering the posttest. Students may have lost some of their learning during the two-week spring break. School offers many students a set routine that they may not otherwise have at home and it is difficult for some students to adjust back to being in school after an extended break. Not only may some students have lost some mathematical learning, but the teachers for both the control and the treatment group had to spend some of their instructional minutes going over procedures and routines once the students returned from break.

Another limitation of this study is during the same time as the researcher was implementing the intervention with the treatment group, the teachers for both the treatment and the control group were attending multiple Professional Development Days (PDs). While both teachers were attending PDs, substitutes were teaching their students math and although both teachers left substitute plans, these substitutes had not been trained on the study or how to ensure fidelity. Additionally, the PDs may have affected the two teachers' math instructional techniques due to the fact that the site administrator encouraged both teachers to try the new teaching methods.

Not only did students miss multiple days of intervention due to a two-week spring break and multiple PD days, but the researcher also attempted to use different assessments for the pretest and posttest. Initially, the researcher used the Eureka Math Mid-Module 5 Assessment for the pretest and the Eureka Math End-of-Module 5 Assessment for the posttest. Using two different assessments for the pretest and the posttest provided invalid results as the two tests measured different skills. Upon realizing this, the researcher administered the Eureka Math Mid-Module 5 Assessment a second time and used the results for the posttest. Due to the fact that administering the Eureka Math Mid-Module 5 Assessment was separated from completing the intervention by about a week, the posttest may have been less valid in measuring the results of the intervention. In the future, it would be ideal for the researcher to administer the same assessment for the pretest and the posttest initially to ensure validity.

One final limitation of this study is that some of the math manipulatives were a preferred activity for the students and some students used the manipulatives as a toy rather than as a tool. When students were using manipulatives at independent workstations, the researcher observed some students using the manipulatives to build structures or construct artistic designs rather than using the manipulatives for mathematical purposes. Additionally, there were numerous times when the resource teacher was in the classroom and specifically removed the manipulatives from resource students that were not appropriately using the manipulatives. For future studies on this subject, researchers may want to consider providing more specific directions on the use of manipulatives and how to structure manipulative use at independent work stations to decrease off-task behavior with the manipulatives.

While there were not enough white students in the sample to look at the comparison in scores between white students and students of color, future research could use a larger sample size to identify if using math manipulatives reduces the opportunity gap for students of color. Future studies on the opportunity gap in mathematics may also want to look at other hands-on strategies other than the use of manipulatives. Researchers could also compare different types of manipulatives to see if some math manipulatives are more helpful for student learning than others. Another idea for future learning would be to study the effect of computer-based manipulatives and how manipulating objects on a computer helps students understand abstract math. Future research could also study the opportunity gap between girls and boys in mathematics.

The purpose of this study was to help students of color improve on math assessments in the field of mathematics through the use of math manipulatives. The researcher used a quantitative, quasi-experimental study with nonequivalent groups, pretest-posttest design. The treatment group used math manipulatives daily while the control group used the scripted curriculum without access to manipulatives. The results of this study showed that using math manipulatives was effective for helping students of color make growth in mathematics.

References

Achieve, I. (2013). Advanced math: Closing the equity Gap. Math Works. Achieve, Inc. Achieve, Inc.

Akiba, M., Letendre, G., & Scribner, J. (2007). Teacher quality, opportunity gap, and national achievement in 46 countries. *Educational Researcher*, *36,* 369-387. doi: 10.3102/0013189x07308739

Anderson, S., Medrich, E., & Fowler, D. (2007). Which achievement gap? *Phi Delta Kappan*, *88*, 547-550. doi:10.1177/003172170708800716

Better, S. J. (2008). *Institutional racism a primer on theory and strategies for social change*. Lanham, MD: Rowman & Littlefield.

Brown v. Board of Education, 347 U.S. 483 (1954). doi:10.1007/978-1-137-07126-2_5

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology, 105*, 380-400. doi:10.1037/a0031084

Coleman, D. (2018). Data Analysis. Unpublished raw data.

Courageous Conversation – News and views about race and equity for educators. (2018). Retrieved from https://courageousconversation.com/

DiAngelo, R. (2016). *What does it mean to be white?: Developing white racial literacy.* New York, NY: Peter Lang. doi:10.3726/978-1-4539-1848-7

Dimitrov, D. M. (2012). *Statistical methods for validation of assessment scale data in counseling and related fields.* Alexandria, VA: American Counseling Association.

Education Data Partnership. (2018). Retrieved from http:/www.ed-data.org/

Fujimura, N. (2001). Facilitating children's proportional reasoning: A model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology, 93*, 589-603. doi:10.1037/0022-0663.93.3.589

Furner, J. M., Yahya, N., & Duffy, M. L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in School and Clinic, 41*(1), 16-23. doi: 10.1177/10534512050410010501

Grade 4 Mathematics. (2014). Retrieved from https://www.engageny.org/resource/grade-4-mathematics

Hemphill, F., Vanneman, A., & Rahman, T. (2011). Achievement gaps: How Hispanic and white students in public schools perform in mathematics and reading on the National Assessment of Educational Progress (Statistical analysis report (National Center for Education Statistics). Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Dept. of Education. doi:10.1037/e595292011-001

Johnson, A. (2016). *Privilege, power and difference*. New York, NY: McGraw-Hill Education.

Katz, J. H. (1990). Some aspects and assumptions of white culture in the United States. Seaside, CA: The Kaleel Jamison Consulting Group, Inc.

Kotok, S. (2017). Unfulfilled potential: High-achieving minority students and the high school achievement gap in math. *High School Journal*, *100*, 183–202. doi:10.1353/hsj.2017.0007

Ladson-Billings, G., & Tate, W. F., IV. (1995). Toward a critical race theory of education. *Teachers College Record*, *97*(1), 47-68.

Levene, H. (1960). Robust tests for equality of variances. *Contributions to Probability and Statistics, 1,* 278-292.

Liggett, R. (2017). The impact of use of manipulatives on the math scores of grade 2 students.

Brock Education Journal, 26(2), 87-101.

McCarthy, M., Eckes, S., Decker, J. (2019). Legal rights of school leaders, teachers, and students (8th Edition). New Jersey: Pearson.

McIntosh, P. (1990). White privilege: Unpacking the invisible knapsack. *In race, class, and gender in the United States* (pp. 172-177). New York, NY: Worth /Macmillan Learning.

McMillan, J. H. (2016). Fundamentals of Educational Research (7th Ed.) Boston, MA: Pearson

Miretzky, D., Chennault, R. E., & Fraynd, D.J. (2016). Closing an opportunity gap: How a modest program made a difference. *Education and Urban Society*, *48*(1), 48-76. doi:10.1177/0013124513501320

National Center for Education Statistics, (2010). *Status and trends in the education of racial and ethnic groups.* Washington, DC: Institute of Education Services, U.S. Department of Education.

Orfield, G., Jongyeon, E., Frankenberg, E., & Siegel-Hawley, G. (2016) Brown at 62: School segregation by race, poverty and state. *Civil Rights Project / Proyecto Derechos Civiles*, UCLA.

Pitre, C. (2014). Improving African American student outcomes: Understanding educational achievement and strategies to close opportunity gaps. *The Western Journal of Black Studies*, *38,* 209-217.

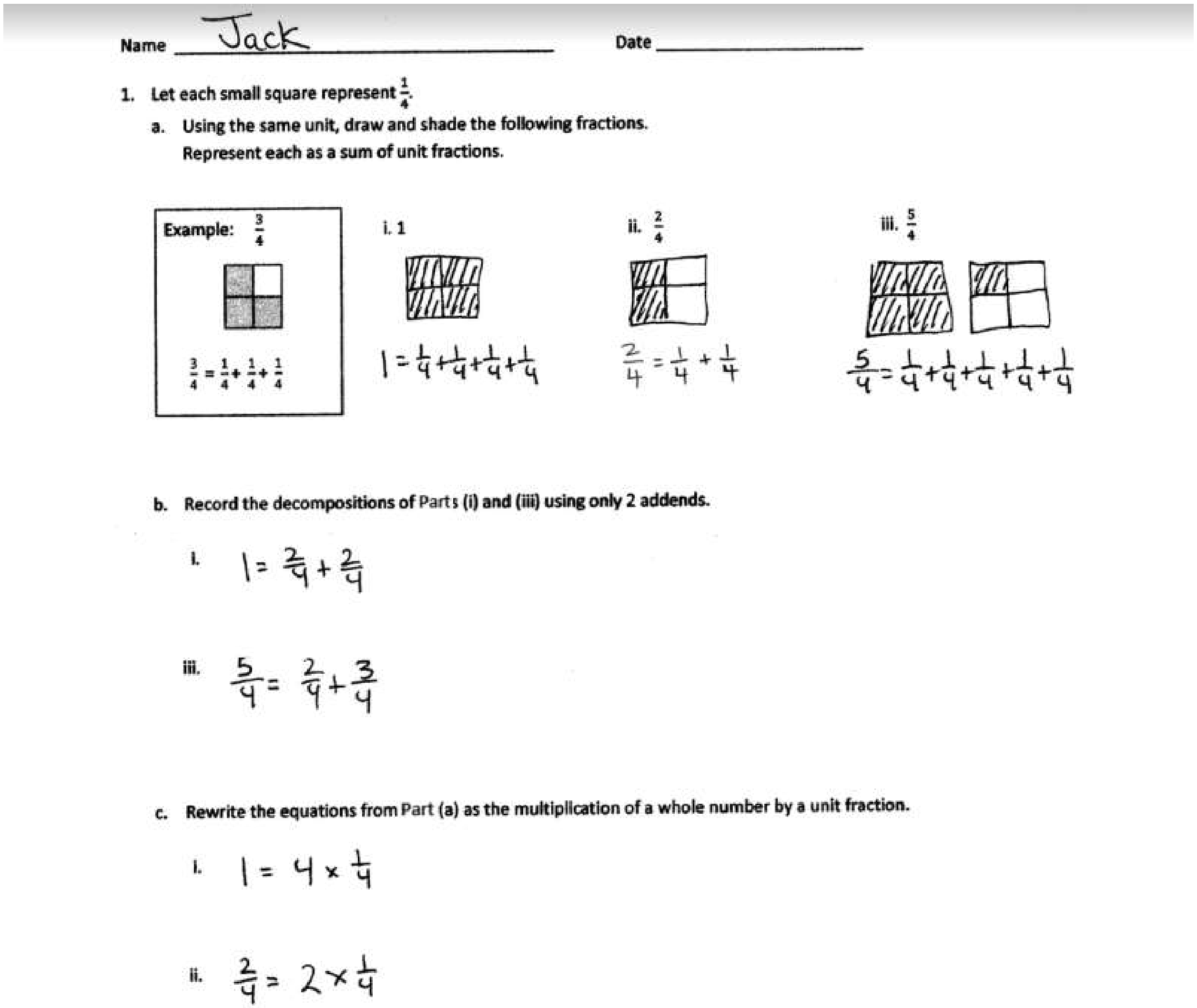
Reardon, S.F., Galindo, C. (2009). The Hispanic-White achievement gap in math and reading in the elementary grades. *American Educational Research Journal, 46,* 853-891. doi: 10.3102/0002831209333184

Rothenberg, P. S. (2010). *Race, class, and gender in the United States: An integrated study*. New York, NY: Worth /Macmillan Learning.

Vanneman, A. (2009). Achievement gaps: How black and white students in public schools perform in mathematics and reading on the National Assessment of Educational Progress (Statistical analysis report (National Center for Education Statistics). Washington, DC: National Center for Education Statistics, Institute of Education Sciences, U.S. Dept. of Education.

Appendix A

Eureka Math Mid-Module 5 Assessment



Name Jack Date

1. Let each small square represent $\frac{1}{4}$.

a. Using the same unit, draw and shade the following fractions. Represent each as a sum of unit fractions.

Example: $\frac{3}{4}$

$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

i. 1

$1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

ii. $\frac{2}{4}$

$\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$

iii. $\frac{5}{4}$

$\frac{5}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

b. Record the decompositions of Parts (i) and (iii) using only 2 addends.

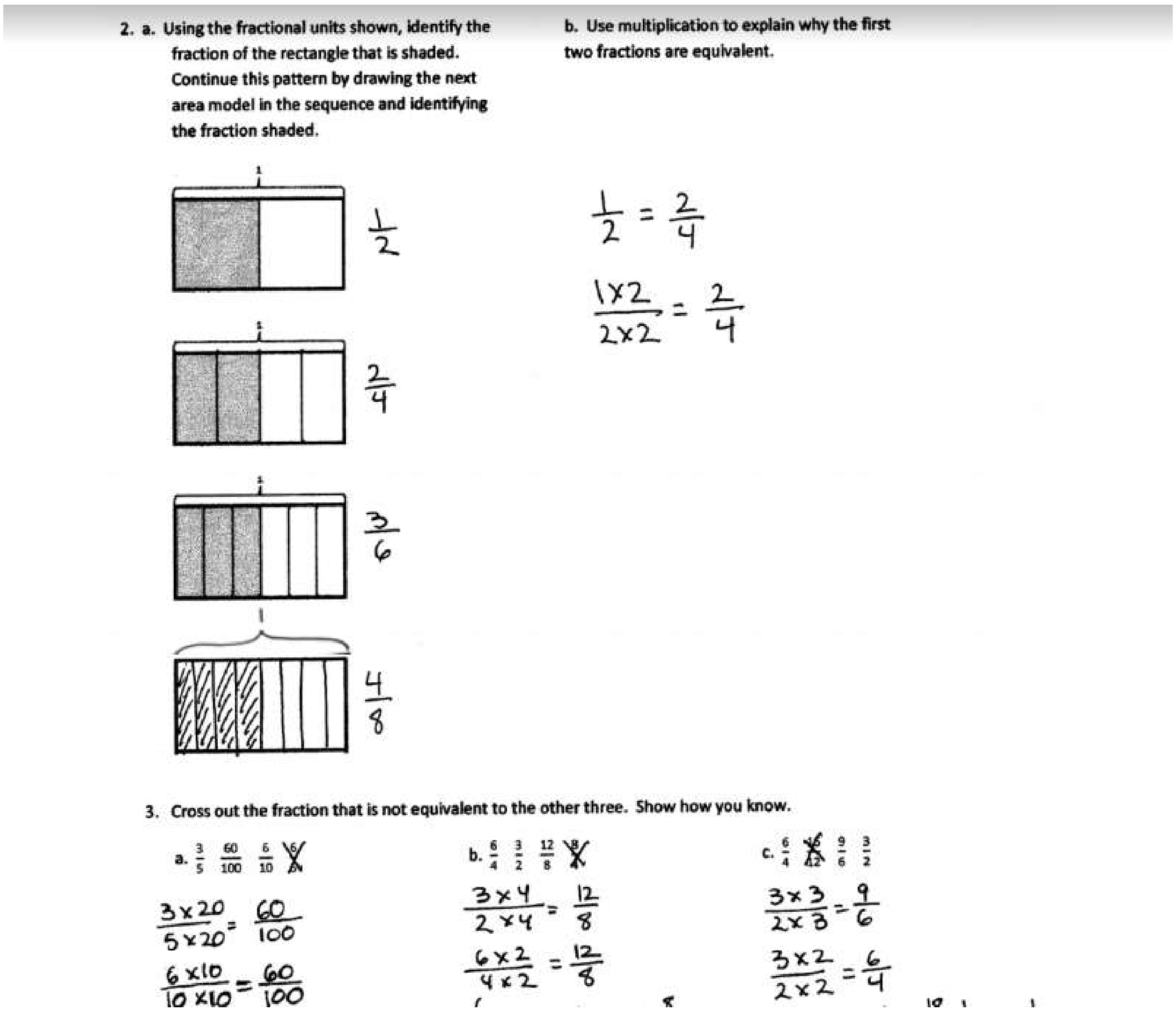
i. $1 = \frac{2}{4} + \frac{2}{4}$

iii. $\frac{5}{4} = \frac{2}{4} + \frac{3}{4}$

c. Rewrite the equations from Part (a) as the multiplication of a whole number by a unit fraction.

i. $1 = 4 \times \frac{1}{4}$

ii. $\frac{2}{4} = 2 \times \frac{1}{4}$



2. a. Using the fractional units shown, identify the fraction of the rectangle that is shaded. Continue this pattern by drawing the next area model in the sequence and identifying the fraction shaded.

$\frac{1}{2}$

$\frac{2}{4}$

$\frac{3}{6}$

$\frac{4}{8}$

b. Use multiplication to explain why the first two fractions are equivalent.

$\frac{1}{2} = \frac{2}{4}$

$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$

3. Cross out the fraction that is not equivalent to the other three. Show how you know.

a. $\frac{3}{5}$ $\frac{60}{100}$ $\frac{6}{10}$ $\cancel{\frac{6}{8}}$

$\frac{3 \times 20}{5 \times 20} = \frac{60}{100}$

$\frac{6 \times 10}{10 \times 10} = \frac{60}{100}$

b. $\frac{6}{4}$ $\frac{3}{2}$ $\frac{12}{8}$ $\cancel{\frac{8}{4}}$

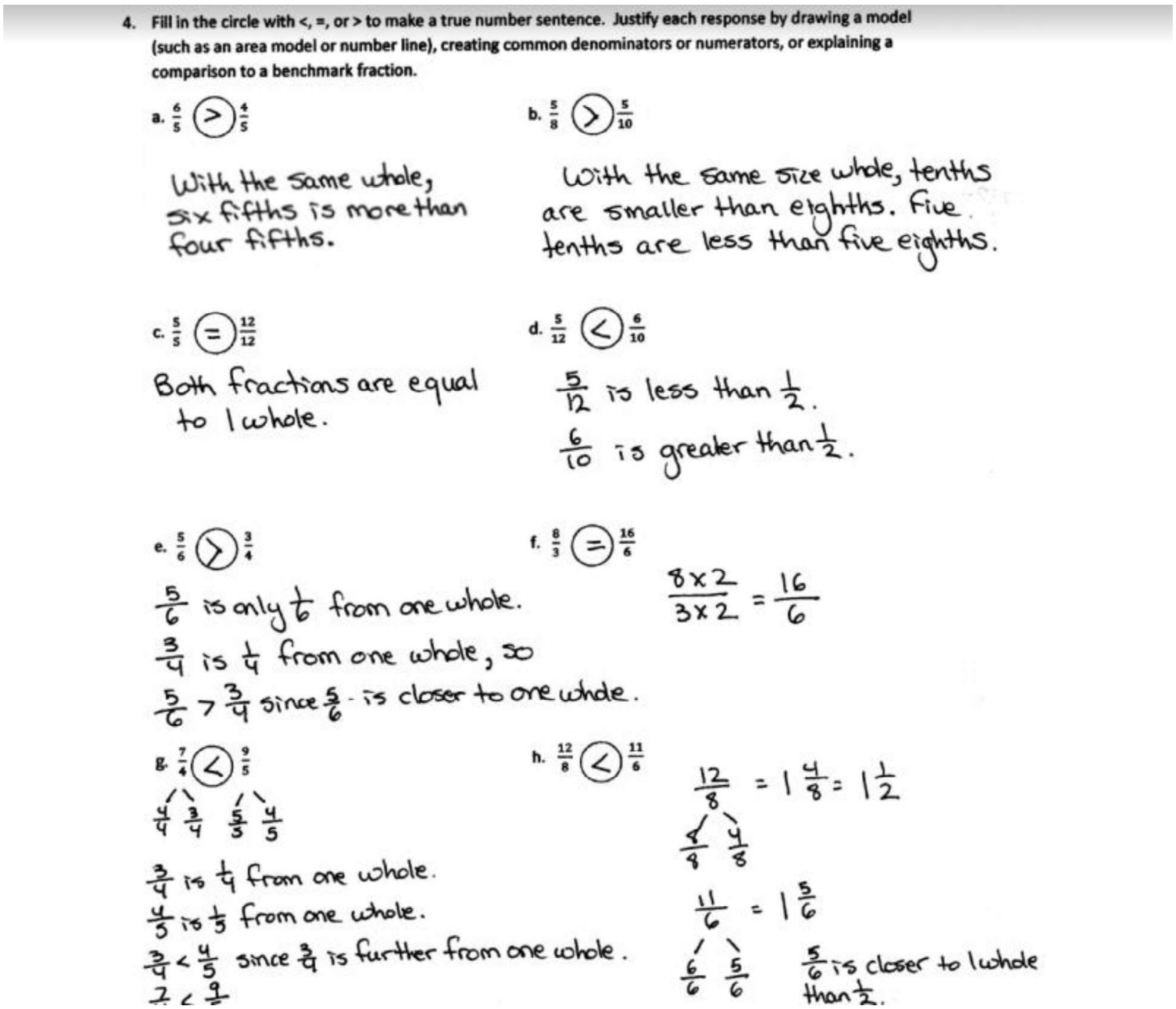
$\frac{3 \times 4}{2 \times 4} = \frac{12}{8}$

$\frac{6 \times 2}{4 \times 2} = \frac{12}{8}$

c. $\frac{6}{4}$ $\cancel{\frac{16}{12}}$ $\frac{9}{6}$ $\frac{3}{2}$

$\frac{3 \times 3}{2 \times 3} = \frac{9}{6}$

$\frac{3 \times 2}{2 \times 2} = \frac{6}{4}$



4. Fill in the circle with <, =, or > to make a true number sentence. Justify each response by drawing a model (such as an area model or number line), creating common denominators or numerators, or explaining a comparison to a benchmark fraction.

a. $\frac{6}{5} > \frac{4}{5}$

With the same whole, six fifths is more than four fifths.

b. $\frac{5}{8} > \frac{5}{10}$

With the same size whole, tenths are smaller than eighths. Five tenths are less than five eighths.

c. $\frac{5}{5} = \frac{12}{12}$

Both fractions are equal to 1 whole.

d. $\frac{5}{12} < \frac{6}{10}$

$\frac{5}{12}$ is less than $\frac{1}{2}$.

$\frac{6}{10}$ is greater than $\frac{1}{2}$.

e. $\frac{5}{6} > \frac{3}{4}$

$\frac{5}{6}$ is only $\frac{1}{6}$ from one whole.

$\frac{3}{4}$ is $\frac{1}{4}$ from one whole, so

$\frac{5}{6} > \frac{3}{4}$ since $\frac{5}{6}$ is closer to one whole.

f. $\frac{8}{3} = \frac{16}{6}$

$$\frac{8 \times 2}{3 \times 2} = \frac{16}{6}$$

g. $\frac{7}{4} < \frac{9}{5}$

$\frac{4}{4}$ $\frac{3}{4}$ $\frac{5}{5}$ $\frac{4}{5}$

$\frac{3}{4}$ is $\frac{1}{4}$ from one whole.

$\frac{4}{5}$ is $\frac{1}{5}$ from one whole.

$\frac{3}{4} < \frac{4}{5}$ since $\frac{3}{4}$ is further from one whole.

$\frac{7}{4} < \frac{9}{5}$

h. $\frac{12}{8} < \frac{11}{6}$

$\frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$

$\frac{8}{8}$ $\frac{4}{8}$

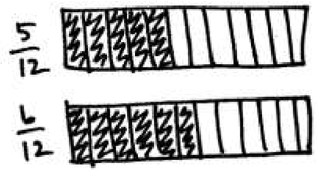
$\frac{11}{6} = 1\frac{5}{6}$

$\frac{6}{6}$ $\frac{5}{6}$

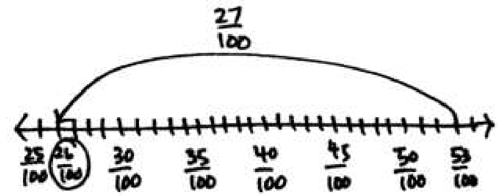
$\frac{5}{6}$ is closer to 1 whole than $\frac{1}{2}$.

5. Fill in the blanks to make each number sentence true. Draw a number line, tape diagram, or area model to represent each problem.

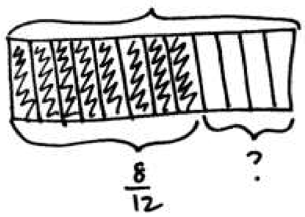
a. $\underline{\frac{11}{12}} = \frac{5}{12} + \frac{6}{12}$



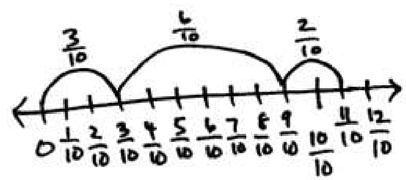
b. $\frac{53}{100} - \frac{27}{100} = \underline{\frac{26}{100}}$



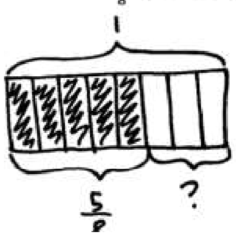
c. $\frac{8}{12} + \underline{\frac{4}{12}} = 1$



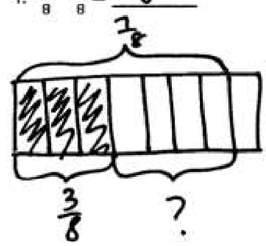
d. $\frac{3}{10} + \frac{6}{10} + \frac{2}{10} = \underline{\frac{11}{10}}$



e. $1 - \frac{5}{8} = \underline{\frac{3}{8}}$ $\frac{8}{8} - \frac{5}{8} = ?$

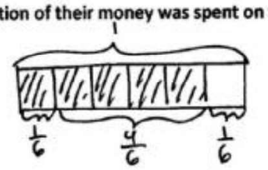


f. $\frac{7}{8} - \frac{3}{8} = \underline{\frac{4}{8}}$



6. Ray, Robin, and Freddy went fishing.

a. They spent $\frac{1}{6}$ of their money on water, $\frac{4}{6}$ of their money on lunch, and the rest on worms. What fraction of their money was spent on worms? Draw a model and write an equation to solve.



$\frac{1}{6} + \frac{4}{6} + \frac{1}{6} = \frac{6}{6} = 1$

They spent $\frac{1}{6}$ of their money on worms.

b. Robin noticed her water bottle was $\frac{1}{2}$ full and Freddy's was $\frac{3}{4}$ full. Robin said, "My $\frac{1}{2}$ full bottle has more water than your $\frac{3}{4}$ full bottle." Explain how $\frac{1}{2}$ bottle could be more than $\frac{3}{4}$ bottle.

If Robin's water bottle was bigger than Freddy's, half of her water bottle could be more than $\frac{3}{4}$ of his.

c. Ray, Robin, and Freddy each had identical containers of worms. Ray used $\frac{3}{8}$ container. Robin used $\frac{6}{8}$ container, and Freddy used $\frac{7}{8}$ container. How many total containers of worms did they use?

$\frac{3}{8} + \frac{6}{8} + \frac{7}{8} = \frac{16}{8} = 2$

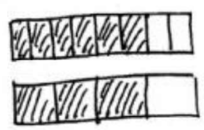
They used 2 containers of worms.

d. Express the number of remaining containers as a product of a whole number and a unit fraction.

$\frac{8}{8} = 8 \times \frac{1}{8}$

e. Six out of the eight fish they caught were trout. What is another fraction equal to 6 eighths? Write a number sentence and draw a model to show the two fractions are equal.

$\frac{6}{8} = \frac{3}{4}$



$\frac{6}{8}$

$\frac{3}{4}$

$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

Appendix B

Eureka Math Mid-Module 5 Assessment Rubric

A Progression Toward Mastery				
Assessment Task Item and Standards Assessed	**STEP 1** **Little evidence of reasoning without a correct answer.** **(1 Point)**	**STEP 2** **Evidence of some reasoning without a correct answer.** **(2 Points)**	**STEP 3** **Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer.** **(3 Points)**	**STEP 4** **Evidence of solid reasoning with a correct answer.** **(4 Points)**
1 **4.NF.3ab** **4.NF.4a**	The student correctly answers fewer than four of the eight parts.	The student correctly answers four or five of the eight parts.	The student correctly answers six or seven of the eight parts.	The student correctly does the following: a. Draws and shades to represent the three given fractions and represents each as a sum of unit fractions: i. $1 = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ ii. $\frac{2}{4} = \frac{1}{4}+\frac{1}{4}$ iii. $\frac{5}{4} = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ b. Records the decomposition using two addends. (Answers may vary.) i. $1 = \frac{3}{4} + \frac{1}{4}$ iii. $\frac{5}{4} = \frac{3}{4} + \frac{2}{4}$ c. Rewrites equations as multiplication of a whole number: i. $1= 4 \times \frac{1}{4}$

A Progression Toward Mastery				
2 **4.NF.1**	The student is unable to correctly complete a majority of the problem.	The student is able to correctly identify the fractions naming the three given models but is unable to complete the next model in the sequence and does not correctly explain equivalence using multiplication.	The student is able to correctly identify the fractions naming the three given models and is able to create the next model, as well as identify the appropriate fraction, but offers an incomplete explanation as to why the first two fractions are equivalent.	The student correctly does the following: a. Identifies the shaded fractions as $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ and creates a correct model to represent $\frac{4}{8}$. b. Uses multiplication to explain why $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent: $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$
3 **4.NF.1**	The student is not able to correctly identify any of the non-equivalent fractions. Explanation or modeling is inaccurate.	The student correctly identifies one of the three non-equivalent fractions. Explanation or modeling is incomplete, or the student does not attempt to show work.	The student correctly identifies two of the three non-equivalent fractions. Explanation or modeling is mostly complete.	The student correctly identifies all three of the non-equivalent fractions and gives complete explanations: a. $\frac{6}{5}$ b. $\frac{8}{4}$ c. $\frac{16}{12}$
4 **4.NF.2**	The student correctly compares three or fewer of the fraction sets with little to no reasoning.	The student correctly compares four or five of the fraction sets with some reasoning.	The student correctly compares six or seven of the fraction sets with solid reasoning. OR The student correctly compares all fraction sets with incomplete reasoning on one or two parts.	The student correctly compares all eight of the fraction sets and justifies all answers using models, common denominators or numerators, or benchmark fractions: a. > b. > c. = d. <

A Progression Toward Mastery				
5 **4.NF.3a**	The student correctly completes two or fewer number sentences and does not accurately use models to represent a majority of the problems.	The student correctly completes three number sentences with some accurate modeling to represent the problems.	The student correctly completes four or five number sentences with accurate modeling to represent problems. OR The student correctly completes all number sentences with insufficient models on one or two problems.	The student correctly completes all six number sentences and accurately models each problem using a number line, a tape diagram, or an area model: a. $\frac{11}{12}$ b. $\frac{26}{100}$ c. $\frac{4}{12}$ d. $\frac{11}{10}$ e. $\frac{3}{8}$ f. $\frac{4}{8}$
6 **4.NF.1** **4.NF.2** **4.NF.3abd** **4.NF.4a**	The student correctly completes fewer than three of the five parts with little to no reasoning.	The student correctly completes three of the five parts, providing some reasoning in Part (a), (b), or (c).	The student correctly completes four of the five parts. OR The student correctly completes all five parts but without solid reasoning in Parts (a), (b), or (c).	The student correctly completes all five of the parts: a. Answers $\frac{1}{6}$ and writes an equation and draws a model. b. Accurately explains through words and/or pictures that the two fractions in question refer to two different-size wholes. The water bottle that is half full could be a larger bottle. c. Answers $\frac{16}{8}$ or 2 containers. d. Answers $\frac{8}{8} = 8 \times \frac{1}{8}$.

Appendix C

Control Group Fidelity Checklist

Week	Date Observed	Initials
March 4 – March 8		
March 11 – March 15		

Observer:______________________________ Date:___________________

Number of students present:________________

Teacher

Is the Eureka Math teacher's edition (TE) visibly present? □Yes □No

Is the teacher is using the scripted curriculum? □Yes □No

Does the teacher have math manipulatives out on student or teacher desks? □Yes □No

Students

Do the students have access to math manipulatives? □Yes □No

Are the students using math manipulatives? □Yes □No

Control Group Fidelity Checklist

Observer:______________________________ Date:___________________

Number of students present:________________

Teacher

Is the Eureka Math teacher's edition (TE) visibly present? □Yes □No

Is the teacher is using the scripted curriculum? □Yes □No

Does the teacher have math manipulatives out on student or teacher desks? □Yes □No

Students

Do the students have access to math manipulatives? □Yes □No

Are the students using math manipulatives? □Yes □No

Treatment Group Fidelity Checklist

Week	Date Observed	Initials
March 4 – March 8		
March 11 – March 15		

Observer:_______________________________ Date:____________________

Number of students present:________________

Teacher

Is the Eureka Math teacher's edition (TE) visibly present? □Yes □No

Is the teacher is using the scripted curriculum? □Yes □No

Does the teacher have math manipulatives out on student or teacher desks? □Yes □No

Students

Do the students have access to math manipulatives? □Yes □No

Are the students using math manipulatives? □Yes □No

Treatment Group Fidelity Checklist

Observer:_______________________________ Date:____________________

Number of students present:________________

Teacher

Is the Eureka Math teacher's edition (TE) visibly present? □Yes □No

Is the teacher is using the scripted curriculum? □Yes □No

Does the teacher have math manipulatives out on student or teacher desks? □Yes □No

Students

Do the students have access to math manipulatives? □Yes □No

Are the students using math manipulatives? □Yes □No